

A FULL-DIVERSITY BEAMFORMING SCHEME IN TWO-WAY AMPLIFIED-AND-FORWARD RELAY SYSTEMS

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This study was supported in part by National Natural Science Foundations of China (Grant Nos. 61222103 and 61072058), the State Major Science and Technology Special Projects (Grant No. 2010ZX03003-003-01), BUPT Excellent Ph.D. Students Foundation (Grant No. CX201211), the Postgraduate Innovation Fund of SICE-BUPT 2011. The work of Z. Ding was supported by the U.K. Engineering and Physical Sciences Research Council under Grant EP/I037423/1.

The paper was submitted on April 1, 2013.

Abstract

Consider a simple two-way relaying channel where two single-antenna sources exchange information via a multiple-antenna relay. To such a scenario, all the existing works which can achieve full diversity order are based on the antenna/relay selection, where the difficulty to design the beamforming lies in the fact that a single beamformer needs to serve two destinations. In this paper, we propose a new full-diversity beamforming scheme which ensures that the relay signals are coherently combined at both destinations. Both analytical and numerical results are provided to demonstrate that this proposed scheme can outperform the existing one based on the antenna selection.

Index Terms

Two-way relay systems, beamforming, network coding, amplify-and-forward, diversity, outage probability.

I. INTRODUCTION

Relaying technique has been considered as an efficient method to extend the coverage and improve the system throughput. Particularly two-way relay systems have drawn a lot of attention, since it can double the spectral efficiency of information exchanging by applying network coding [1]–[3]. By broadcasting the mixture of both source messages at the relay, the communication between two sources can be accomplished in two time slots, where each source recovers its desired message by subtracting self-interference from its observation. Compared with the traditional 4 time-slot schemes, network coding can increase the throughput significantly.

To further improve the transmission performance, multiple-input and multiple-output (MIMO) techniques have been introduced to two-way relay channels [4]–[7]. By properly designing the beamforming/precoding, multiple data streams can be transmitted simultaneously in MIMO systems, which provides another reliable approach to improve the performance of relay transmissions. Specifically the performance analysis of network coding in two-way amplified-and-forward (AF) relay systems was derived in [4]. A joint source and relay precoding design for two-way AF relay systems was studied in [5], where the multiplexing gain can be maximized by channel parallelization. In [6], an interference alignment scheme is proposed for multiple two-way relay systems, and the inter-stream interference is eliminated by applying precoding, while the intra-stream interference is coped with by using network coding.

Although there have been such extensive studies about two-way relaying channels, some challenging issues are still left as open problems. For example, consider a simple two-way relaying channel where two single-antenna sources exchange information via a multiple-antenna relay. To such a scenario, all the existing works which can achieve full-diversity gain are based on the relay/antenna selection, and to the best knowledge of the authors, how to design a full diversity relay beamformer is still an open problem. The motivation to design the beamforming is analogue to the performance gain of maximum ratio combining (MRC) over the selection detection in single-input multiple-output (SIMO) systems. However, in the context of two-way relaying, the difficulty to design the beamforming lies in the fact that a single relay beamformer needs to serve both destinations. In this paper, we focus on a simple two-way relaying

scenario and the main contribution of this paper can be summarized as follows:

Firstly, a full-diversity relay beamformer is designed for the addressed scenario, to which the conventional beamforming/precoding techniques, such as the generalized singular value decomposition (GSVD)-based precoding in [5], cannot provide any type of spatial transmission gain. The key idea of our proposed scheme is to utilize the symmetry of the observation phase at two destinations. As a result, the relay signals can be coherently combined at both destinations. Just like the performance gain of MRC over the selective detection, the proposed beamforming scheme can achieve the same diversity gain as the existed antenna selection scheme in [2], [5], and also offer better reception reliability in the low signal-to-noise ratio (SNR) region.

Secondly, to evaluate the performance of the proposed transmission scheme, the outage probability is analyzed in this paper. Particularly an upper bound of SNR is first developed, which facilitates the development of a closed-form expression of outage probability. The asymptotic analysis of such an bound is also provided to demonstrate the full-diversity gain achieved by the proposed scheme. *Finally*, the simulation of outage probability shows that our proposed scheme outperforms the antenna selection scheme, especially when the SNR is low. In addition, the numerical results also demonstrate the accuracy of the derived analytical results.

The rest of this paper is organized as follows. Section II describes the system model, and introduces the proposed full-diversity transmission scheme. In Section III, the performance analysis of proposed scheme is studied. The simulation results are shown in Section IV, and followed by the conclusions in Section V.

Notation: Vectors are denoted as boldface small letters, i.e., \mathbf{a} , a_m denotes the m -th element of \mathbf{a} and \mathbf{a}^T is the transpose of \mathbf{a} . $\|c\|$ is the Frobenius norm of c , and c can be either a vector or a number. $\mathbb{E}\{x\}$ is the expectation of the variable x , and $\Pr\{x < p\}$ denotes the probability that the value of the random variable x is less than p . $\Gamma(d)$ is the Gamma function, $\gamma(a, x)$ denotes the lower incomplete Gamma function, $K_\nu(z)$ is the modified Bessel function with imaginary argument, and all the referred special functions follows the form given in [10].

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

Consider a two-way relay system, where two sources S_1 and S_2 exchange messages via a relay R . As illustrated in Fig. 1, each source node is equipped with one antenna, while the relay is equipped with N antennas. All nodes are assumed to employ the time division duplexing for simplicity. Due to the symmetry of time division duplex systems, the incoming channel and the corresponding outgoing channel are symmetric. All the channels are modeled as quasi-static Rayleigh fading channels, and the relay has the access to full source-relay channel state information (CSI).

The transmission can be accomplished in two phases by applying network coding. During the first phase, both sources transmit their own messages to the relay simultaneously, and the network coded observation at the relay can be expressed as

$$\mathbf{y}_R = \mathbf{h}x_1 + \mathbf{g}x_2 + \mathbf{n}_R, \quad (1)$$

where x_1 is the message transmitted by S_1 , and its transmit power is limited as $\mathbb{E}\{x_1^T x_1\} = P_1$, \mathbf{h} denotes the $N \times 1$ channel vector for the link from S_1 to R , x_2 and \mathbf{g} are defined similarly for S_2 , and the corresponding transmit power is P_2 , and \mathbf{n}_R is the additive Gaussian noise at the relay. Assuming that the AF strategy is applied, the relay broadcasts its network coding message after beamforming in the second phase, and the observations at S_1 and S_2 can be expressed as follows respectively,

$$y_1 = \beta \mathbf{h}^T \mathbf{Q} \mathbf{h} x_1 + \beta \mathbf{h}^T \mathbf{Q} \mathbf{g} x_2 + \beta \mathbf{h}^T \mathbf{Q} \mathbf{n}_R + \mathbf{n}_1, \quad (2)$$

$$y_2 = \beta \mathbf{g}^T \mathbf{Q} \mathbf{g} x_2 + \beta \mathbf{g}^T \mathbf{Q} \mathbf{h} x_1 + \beta \mathbf{g}^T \mathbf{Q} \mathbf{n}_R + \mathbf{n}_2, \quad (3)$$

where \mathbf{Q} is the beamforming matrix at the relay, $\beta = \sqrt{\frac{P_R}{|\mathbf{Q}\mathbf{h}|^2 P_1 + |\mathbf{Q}\mathbf{g}|^2 P_2 + |\mathbf{Q}|^2 \sigma^2}}$ is the power normalization factor at the relay, P_R is the relay transmit power and \mathbf{n}_i is the additive Gaussian noise at S_i , $i = 1, 2$. By using the global CSI, S_i can subtract its self-interference from the observed network coding message, and obtains the desired message from the other source.

A. Full-Diversity Beamforming Design for Studied Two-Way AF Relay Systems

Since the relay is with multiple antennas, antenna selection is a straightforward method to achieve full diversity gain, and improves the transmission performance. On the other hand, when all the relay antennas are utilized, the conventional beamforming/precoding schemes, such as the generalized singular

value decomposition (GSVD) based scheme in [5], cannot achieve the best diversity performance, which is due to the fact that the signal space at the sources is flattened into one dimension. To provide a full-diversity gain scheme with better performance, the beamforming design is proposed in this subsection. First the definition of diversity is introduced as follows,

Definition 1: *Spacial diversity* can be achieved by redundantly receiving the same message over independent fading channels. Particularly the diversity gain is α if the transmission scheme satisfies that

$$\lim_{\rho \rightarrow \infty} \frac{P^{\text{out}}}{1/\rho} = \alpha \quad (4)$$

where P^{out} describes the probability that the transmission fails, as defined in [8], and ρ denotes the average SNR.

To have a closer look at the diversity order, we further derive the definition of outage probability based on the results given in [8],

$$P^{\text{out}} = \Pr\{R < R_{\text{th}}\} = \Pr\{\text{SNR} < \gamma_{\text{th}}\}, \quad (5)$$

where R is the transmission data rate, R_{th} is set as the threshold for R , SNR is the receive SNR and γ_{th} is its threshold. The last equation shows that both the outage probability and the diversity gain are mainly determined by the receive SNR. Recalling the observations given in (2) and (3), the SNRs for S_1 and S_2 can be derived as follows respectively,

$$\text{SNR}_1 = \frac{\beta^2 |\mathbf{h}^T \mathbf{Q} \mathbf{g}|^2 P}{(\beta^2 |\mathbf{h}^T \mathbf{Q}|^2 + 1) \sigma^2} = \frac{|\mathbf{h}^T \mathbf{Q} \mathbf{g}|^2}{2 |\mathbf{Q} \mathbf{h}|^2 + |\mathbf{Q} \mathbf{g}|^2 + |\mathbf{Q}|^2 / \rho} \rho, \quad (6)$$

$$\text{SNR}_2 = \frac{\beta^2 |\mathbf{g}^T \mathbf{Q} \mathbf{h}|^2 P}{(\beta^2 |\mathbf{g}^T \mathbf{Q}|^2 + 1) \sigma^2} = \frac{|\mathbf{g}^T \mathbf{Q} \mathbf{h}|^2}{2 |\mathbf{Q} \mathbf{g}|^2 + |\mathbf{Q} \mathbf{h}|^2 + |\mathbf{Q}|^2 / \rho} \rho, \quad (7)$$

where the transmit power is set as $P_R = P_1 = P_2 = P$ for simplicity, and the average SNR $\rho = P/\sigma^2$.

Limited by the single antenna setting at the source nodes, the key step to achieve full diversity gain is to design the relay beamforming matrix \mathbf{Q} . As introduced in [9], the SNRs of full-diversity schemes share the common structure as follows. Over a $K \times 1$ channel with the fading vector \mathbf{f} ,

$$\text{SNR}^{\text{ful-div}} = |\mathbf{f}|^2 \rho = \sum_{k=1}^K |f_k|^2 \rho. \quad (8)$$

Such a result shows that *the diversity gains can be achieved by coherently accumulating the signal power from different pathes at the destination*, which coincides with the clarification in Definition 1. Then we focus on the signal parts in the SNRs of S_1 and S_2 ,

$$\mathbf{h}^T \mathbf{Q} \mathbf{g} = \sum_{m=1}^N \sum_{n=1}^N q_{mn} h_m g_n = \sum_{m=1}^N \sum_{n=1}^N q_{mn} |h_m| |g_n| e^{j(\phi_m + \theta_n)}, \quad (9)$$

$$\mathbf{g}^T \mathbf{Q} \mathbf{h} = \sum_{m=1}^N \sum_{n=1}^N q_{mn} g_m h_n = \sum_{m=1}^N \sum_{n=1}^N q_{mn} |g_m| |h_n| e^{j(\theta_m + \phi_n)}, \quad (10)$$

where ϕ_p denotes the argument of h_p and θ_q is defined similarly for g_q . As shown in (9) and (10), the uncertainty of arguments blocks the reception enhancement at each source node. To ensure the beamforming design can benefit both sources, the beamforming matrix can be defined as

$$\mathbf{Q} = (q_{mn})_{N \times N} = \begin{cases} e^{-j(\phi_m + \theta_n)}, & m = n \\ 0, & m \neq n \end{cases}. \quad (11)$$

Substituting (11) into (6) and (7), the SNRs at S_1 and S_2 can be further derived as

$$\text{SNR}_1 = \frac{(\sum_{m=1}^N |h_m| |g_m|)^2}{2|\mathbf{h}|^2 + |\mathbf{g}|^2 + N/\rho} \rho, \quad \text{SNR}_2 = \frac{(\sum_{m=1}^N |h_m| |g_m|)^2}{2|\mathbf{g}|^2 + |\mathbf{h}|^2 + N/\rho} \rho. \quad (12)$$

By adjusting the argument of channel fading for each path, the receive signal power can be strengthen at both sources, and thus the full-diversity gain can be achieved, which is demonstrated in the next section.

III. PERFORMANCE ANALYSIS FOR THE PROPOSED FULL-DIVERSITY TRANSMISSION SCHEME

In this section, the performance of the proposed full-diversity transmission scheme is evaluated. Firstly, a tractable upper bound of outage probability is derived, and then its closed-form expression can be obtained. Next the asymptotical high SNR approximation of the derived upper bound is analyzed, which demonstrates the diversity gain of the proposed transmission scheme.

A. Upper Bound of Outage Probability for the Proposed Scheme

Recalling the definition given in (5), the outage probability can be derived by applying the probability density function (PDF) of SNR. Since the channel vectors for both links are coupled tightly in the signal parts of SNR_1 and SNR_2 , it is hard to derive their PDFs. To find a feasible method to evaluate the outage probability of the proposed transmission scheme, we first derive a tractable upper bound for the SNRs, and the following lemma is presented.

Lemma 1: Denoting that $w = \frac{(\sum_{m=1}^N |h_m| |g_m|)^2}{\frac{1}{N} (\sum_{m=1}^N |h_m|^2) (\sum_{n=1}^N |g_n|^2)}$, it can be bounded as follows. When the number of relay antennas N is large enough,

$$w \stackrel{N \rightarrow \infty}{\geq} 1. \quad (13)$$

Proof: We first further derive w as

$$w = \frac{\frac{1}{N} (\sum_{m=1}^N |h_m|^2 |g_m|^2 + 2 \sum_{m \neq n} |h_m| |g_m| |h_n| |g_n|)}{(\frac{1}{N} \sum_{m=1}^N |h_m|^2) (\frac{1}{N} \sum_{m=1}^N |g_m|^2)}. \quad (14)$$

As introduced previously, h_m and g_n are independent Gaussian distributed variables. Based on the law of large numbers, the denominator of w in (14) can be approximated as follows,

$$\left(\frac{1}{N} \sum_{m=1}^N |h_m|^2\right) \left(\frac{1}{N} \sum_{n=1}^N |g_n|^2\right) \stackrel{N \rightarrow \infty}{\approx} [\mathbb{E}(|h_m|^2) + \delta_1] [\mathbb{E}(|g_n|^2) + \delta_2] = (1 + \delta_1)(1 + \delta_2), \quad (15)$$

where δ_1 and δ_2 approaches zero when N goes to infinity. Similarly, both parts of the numerator in (14) can be given as follows when N approaches infinity,

$$\frac{1}{N} \sum_{m=1}^N |h_m|^2 |g_m|^2 \stackrel{N \rightarrow \infty}{\approx} \mathbb{E}(|h_m|^2 |g_m|^2) + \delta_3 = 1 + \delta_3, \quad (16)$$

$$\frac{2}{N} \sum_{m \neq n} |h_m| |g_m| |h_n| |g_n| \stackrel{N \rightarrow \infty}{\approx} 2(N-1) \mathbb{E}(|h_m| |g_m| |h_n| |g_n|) + \delta_4 = \frac{1}{2}(N-1)\pi^2 + \delta_4, \quad (17)$$

where the last equation follows the fact that $|h_p|$ and $|g_q|$ are independent Rayleigh distribution variables, and δ_3 and δ_4 are infinitesimals. Substituting (15), (16) and (17) into (14), we can obtain that

$$w \stackrel{N \rightarrow \infty}{\approx} \frac{1 + \frac{1}{2}(N-1)\pi^2 + \delta_3 + \delta_4}{1 + \delta_1 + \delta_2 + \delta_1\delta_2} = 1 + \frac{1}{2}(N-1)\pi^2 \geq 1. \quad (18)$$

The lemma has been proved. ■

Based on Lemma 1, the lower bound of the receive SNRs can be given as follows. When the number of relay antennas N is large, the channel vectors can be decoupled at the signal parts for both sources,

$$\text{SNR}_1 \geq \frac{|\mathbf{h}|^2 |\mathbf{g}|^2}{N(2|\mathbf{h}|^2 + |\mathbf{g}|^2 + N/\rho)} \rho, \quad \text{SNR}_2 \geq \frac{|\mathbf{h}|^2 |\mathbf{g}|^2}{N(2|\mathbf{g}|^2 + |\mathbf{h}|^2 + N/\rho)} \rho. \quad (19)$$

Then an upper bound of outage probability can be derived in closed-form, which is presented in the next theorem.

Theorem 2: The upper bound of outage probability for the proposed scheme in Section II can be expressed as

$$\begin{aligned} P^{\text{out-up}} &= \frac{\gamma(N, \frac{2N\gamma_{\text{th}}}{\rho})}{\Gamma(N)} + \sum_{j=0}^{N-1} \binom{N-1}{j} \frac{(2N\gamma_{\text{th}})^j}{\Gamma(N)} \Gamma\left[N-j, \frac{(N-j)b}{\rho}\right] \frac{e^{-2Na}}{\rho^j} - \\ &\quad \sum_{i=0}^{N-1} \sum_{k=0}^i \sum_{l=0}^{N-1} 2 \binom{i}{k} \binom{N-1}{l} \frac{(2N)^l b^{\frac{N+k-l}{2}} \gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)} K_{k+l-N} \left(\frac{2}{\rho} \sqrt{b}\right) \frac{e^{-\frac{(2N+1)\gamma_{\text{th}}}{\rho}}}{\rho^{N+i}}, \end{aligned} \quad (20)$$

where $a = \frac{\gamma_{\text{th}}}{\rho}$ and $b = N\gamma_{\text{th}}(1 + 2\gamma_{\text{th}})$.

Proof: Without loss of the generality, the upper bound of outage probability for S_1 is derived. Denoting that $x_1 = |\mathbf{h}|^2$ and $x_2 = |\mathbf{g}|^2$, it can be given as

$$\begin{aligned} P^{\text{out-up}} &= \Pr \left\{ \frac{x_1 x_2}{N(2x_1 + x_2 + N/\rho)} \rho < \gamma_{\text{th}} \right\} \\ &= \Pr \{x_2 < 2Na\} + \Pr \left\{ 0 < x_1 < \frac{\gamma_{\text{th}} x_2 + Na}{\rho x_2 - 2N\gamma_{\text{th}}}, x_2 > 2Na \right\} = \mathcal{K}_1 + \mathcal{K}_2. \end{aligned} \quad (21)$$

As introduced previously, x_1 and x_2 are two independent Chi-squared distribution variables, whose PDFs can be shown as

$$f_{x_1} = \frac{x_1^{N-1} e^{-x_1}}{\Gamma(N)}, f_{x_2} = \frac{x_2^{N-1} e^{-x_2}}{\Gamma(N)}. \quad (22)$$

Then substituting (22) into (21), \mathcal{K}_1 and \mathcal{K}_2 can be obtained,

$$\mathcal{K}_1 = \int_0^{2Na} f_{x_2} dx_2 = \int_0^{2Na} \frac{x_2^{N-1} e^{-x_2}}{\Gamma(N)} dx_2 = \frac{\gamma(N, 2Na)}{\Gamma(N)}, \quad (23)$$

$$\begin{aligned} \mathcal{K}_2 &= \int_{2Na}^{\infty} f_{x_2} \left(\int_0^{\frac{\gamma_{\text{th}} x_2 + Na}{\rho x_2 - 2N\gamma_{\text{th}}}} \frac{x_1^{N-1} e^{-x_1}}{\Gamma(N)} dx_1 \right) dx_2 \\ &= \frac{1}{[\Gamma(N)]^2} \int_{2Na}^{\infty} \gamma \left(N, \frac{\gamma_{\text{th}} x_2 + Na}{\rho x_2 - 2N\gamma_{\text{th}}} \right) x_2^{N-1} e^{-x_2} dx_2. \end{aligned} \quad (24)$$

To obtain the closed-form expression, (24) can be expanded as follows by denoting that $z = \frac{\gamma_{\text{th}} x_2 + Na}{\rho x_2 - 2N\gamma_{\text{th}}}$,

$$\begin{aligned} \mathcal{K}_2 &= \frac{b}{[\Gamma(N)]^2} \int_a^{\infty} \frac{(2N\gamma_{\text{th}}z + Na)^{N-1}}{(\rho z - \gamma_{\text{th}})^{N+1}} \gamma(N, z) e^{-\frac{2N\gamma_{\text{th}}z + Na}{\rho z - \gamma_{\text{th}}}} dz \\ &= \frac{b}{\Gamma(N)} \int_a^{\infty} \frac{(2N\gamma_{\text{th}}z + Na)^{N-1}}{(\rho z - \gamma_{\text{th}})^{N+1}} e^{-\frac{2N\gamma_{\text{th}}z + Na}{\rho z - \gamma_{\text{th}}}} dz - \\ &\quad \frac{b}{\Gamma(N)} \sum_{i=0}^{N-1} \int_a^{\infty} \frac{z^i (2N\gamma_{\text{th}}z + Na)^{N-1}}{\Gamma(i) (\rho z - \gamma_{\text{th}})^{N+1}} e^{-\left(\frac{2N\gamma_{\text{th}}z + Na}{\rho z - \gamma_{\text{th}}} + z\right)} dz = \mathcal{L}_1 - \mathcal{L}_2, \end{aligned} \quad (25)$$

where \mathcal{K}_2 is simplified by applying $\gamma(N, z) = \Gamma(N) [1 - e^{-z} (\sum_{i=1}^{N-1} \frac{z^i}{\Gamma(i)})]$ in [10]. Then we focus on deriving the closed-form expression of \mathcal{L}_1 and \mathcal{L}_2 respectively. Particularly denoting that $t = \rho z - \gamma_{\text{th}}$,

\mathcal{L}_1 can be derived as

$$\begin{aligned} \mathcal{L}_1 &= \frac{b e^{-2Na}}{\Gamma(N) \rho^N} \int_0^{\infty} \frac{(2N\gamma_{\text{th}}t + b)^{N-1}}{t^{N+1}} e^{-\frac{b}{\rho t}} dt = \sum_{j=0}^{N-1} \binom{N-1}{j} (2N\gamma_{\text{th}})^j b^{N-j} \frac{e^{-2Na}}{\Gamma(N) \rho^N} \int_0^{\infty} \frac{e^{-\frac{b}{\rho t}}}{t^{N-j+1}} dt \\ &= \sum_{j=0}^{N-1} \binom{N-1}{j} \frac{(2N\gamma_{\text{th}})^j}{\Gamma(N)} \Gamma \left[N-j, \frac{(N-j)b}{\rho} \right] \frac{e^{-2Na}}{\rho^j}, \end{aligned} \quad (26)$$

where \mathcal{L}_1 is simplified by using Binomial theorem. Following the notations in (26), the closed-form expression of \mathcal{L}_2 can be obtained in a similar way,

$$\begin{aligned}
\mathcal{L}_2 &= \sum_{i=0}^{N-1} \frac{b e^{-\frac{(2N+1)\gamma_{\text{th}}}{\rho}}}{\Gamma(N)\Gamma(i)\rho^{N+i}} \int_0^\infty \frac{(t + \gamma_{\text{th}})^i (2N\gamma_{\text{th}}t + b)^{N-1}}{t^{N+1}} e^{-\left(\frac{b}{\rho t} + \frac{t}{\rho}\right)} dt \\
&= \sum_{i=0}^{N-1} \sum_{k=0}^i \sum_{l=0}^{N-1} \binom{i}{k} \binom{N-1}{l} \frac{(2N)^l b^{N-l} \gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)} \frac{e^{-\frac{(2N+1)\gamma_{\text{th}}}{\rho}}}{\rho^{N+i}} \int_0^\infty t^{k+l-N-1} e^{-\left(\frac{b}{\rho t} + \frac{t}{\rho}\right)} dt \\
&= \sum_{i=0}^{N-1} \sum_{k=0}^i \sum_{l=0}^{N-1} 2 \binom{i}{k} \binom{N-1}{l} \frac{(2N)^l b^{\frac{N+k-l}{2}} \gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)} K_{k+l-N} \left(\frac{2}{\rho} \sqrt{b} \right) \frac{e^{-\frac{(2N+1)\gamma_{\text{th}}}{\rho}}}{\rho^{N+i}}. \tag{27}
\end{aligned}$$

Then the proof has been finished. ■

Although the derivation of $P^{\text{out-up}}$ is based on the assumption that the number of relay antennas is large, the simulation results in the next section show that the derived upper bound is a general expression, i.e., $P^{\text{out-up}}$ is also applicable when $N = 2$.

B. Analysis of Diversity Gain for the Proposed Scheme

To derive the achievable diversity gain of the proposed scheme, the asymptotical analysis of $P^{\text{out-up}}$ is given. Specifically it can be obtained by applying high SNR approximation on the original expression of $P^{\text{out-up}}$ given in (21), and the derivation is given in detail as follows,

1) *High SNR Approximation of \mathcal{K}_1 in (21)*: For high SNR region, $\frac{2N\gamma_{\text{th}}}{\rho}$ approaches 0, and thus the lower incomplete gamma function in \mathcal{K}_1 can achieve the following asymptotic form [10],

$$\gamma\left(N, \frac{2N\gamma_{\text{th}}}{\rho}\right) \rightarrow \frac{(2N\gamma_{\text{th}})^N}{N} \frac{1}{\rho^N}. \tag{28}$$

Then the high SNR approximation of \mathcal{K}_1 can be derived as

$$\mathcal{K}_1^\infty = \frac{(2N\gamma_{\text{th}})^N}{N\Gamma(N)} \frac{1}{\rho^N}. \tag{29}$$

2) *High SNR Approximation of \mathcal{K}_2 in (21)*: To derive an accurate approximation, we begin with the expression of \mathcal{K}_2 given in (24). When ρ is in the high SNR region, the argument of the lower incomplete Gamma function in (24) approaches

$$\frac{\gamma_{\text{th}}x_2 + (N\gamma_{\text{th}}/\rho)}{\rho x_2 - 2N\gamma_{\text{th}}} = \frac{\gamma_{\text{th}}x_2 + (N\gamma_{\text{th}}/\rho)}{\rho(x_2 - (2N\gamma_{\text{th}}/\rho))} \rightarrow \frac{\gamma_{\text{th}}}{\rho}. \tag{30}$$

Then the approximation of \mathcal{K}_2 can be derived as follows by substituting (30) into (24),

$$\mathcal{K}_2^\infty = \frac{1}{[\Gamma(N)]^2} \gamma\left(N, \frac{\gamma_{\text{th}}}{\rho}\right) \int_{\frac{2N\gamma_{\text{th}}}{\rho}}^{\infty} x_2^{N-1} e^{-x_2} dx_2 = \frac{1}{[\Gamma(N)]^2} \gamma\left(N, \frac{\gamma_{\text{th}}}{\rho}\right) \left[1 - \gamma\left(N, \frac{2N\gamma_{\text{th}}}{\rho}\right)\right]. \quad (31)$$

Similar to the approximation of \mathcal{K}_1 , the asymptotic form of \mathcal{K}_2 can be given as follows,

$$\mathcal{K}_2^\infty = \frac{\gamma_{\text{th}}^N}{N[\Gamma(N)]^2} \left[1 - \frac{(2N\gamma_{\text{th}})^N}{N} \frac{1}{\rho^N}\right] \frac{1}{\rho^N} = \frac{\gamma_{\text{th}}^N}{N[\Gamma(N)]^2} \frac{1}{\rho^N} + o\left(\frac{1}{\rho^N}\right). \quad (32)$$

Based on the high SNR approximation of \mathcal{K}_1 and \mathcal{K}_2 , the asymptotic form of $P^{\text{out-up}}$ can be given in the following corollary, which demonstrates the diversity gain of the proposed scheme.

Corollary 3: For the high SNR region, the derived upper bound of outage probability for the proposed scheme, which is provided in Theorem 2, can be approximated as

$$P_\infty^{\text{out-up}} = \frac{(2N\gamma_{\text{th}})^N}{N\Gamma(N)} \frac{1}{\rho^N} + \frac{\gamma_{\text{th}}^N}{N[\Gamma(N)]^2} \frac{1}{\rho^N} + o\left(\frac{1}{\rho^N}\right). \quad (33)$$

Substituting (33) into (4), it is easy to show that the diversity gain of the proposed scheme is N , which is the full-diversity for the studied scenario.

IV. NUMERICAL RESULTS

In this section, the numerical results are provided to evaluate the transmission performance and demonstrate the accuracy of theoretical analysis in the previous section. In Fig. 2, the cumulative distribution functions (CDF) of w defined in Lemma 1 are plotted when the number of relay antennas is set as $N = 3, 4, 5, 6$. As shown in the figure, $\Pr\{w > 1\}$ raises as N increases, and approaches 1 when $N \geq 3$. Such results verify the asymptotic inequality given in Lemma 1.

Fig. 3 gives the numerical results for the derived upper bound of outage probability and the corresponding asymptotic analysis, where N is set as 2, 3 and 4. The simulation results show that the derived upper bound $P^{\text{out-up}}$ is quite tight in the high SNR region, especially when N is small. Moreover, the outage probability of the proposed scheme decreases faster when the relay is equipped with more antennas, which demonstrates the diversity gain. The curves for the upper bound and the asymptotic analysis have the same slope as the Monte-Carlo curve, and thus the analysis results in Section III can be testified.

To further evaluate the outage performance of the proposed scheme, the comparison with the antenna selection scheme and the direct AF scheme for the studied scenario is provided in Fig. 4, and the numbers of relay antennas are given as $N = 2, 3, 4$, respectively. Since the direct AF scheme transmits the messages

without additional operations, it does not obtain any diversity gain, while both the proposed scheme and the antenna selection scheme can achieve full-diversity gain. As shown in the figure, the proposed scheme can achieve better performance than the antenna selection scheme, especially in the low SNR region. Compared with the antenna selection scheme, our proposed scheme can reduce the required SNR by 1.3 dB with $N = 3$ and when the outage probability achieves 10^{-2} , and such performance gain can be enlarged as N increases.

V. CONCLUSION

To achieve full-diversity gain, a joint beamforming and network coding scheme was proposed for two-way relay systems. By combining the messages from different paths at the sources, the transmission reliability can be improved. Then a closed-form upper bound of outage probability was derived for the proposed scheme, and the high SNR asymptotic analysis was also given to demonstrate the achieved diversity gain. The simulation results were provided to verify the derived analysis results, which also shows that our proposed scheme outperforms antenna selection scheme in the studied scenario.

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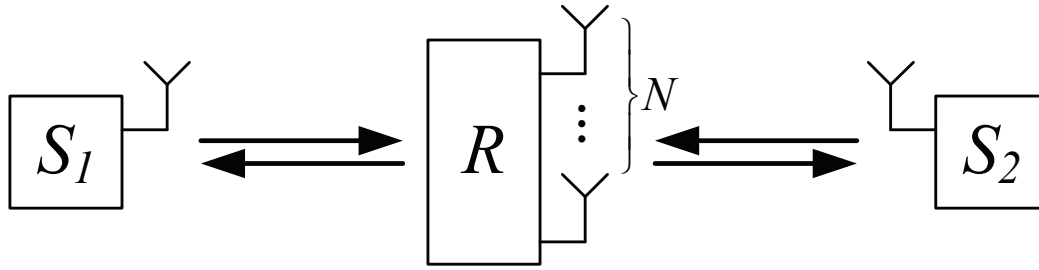
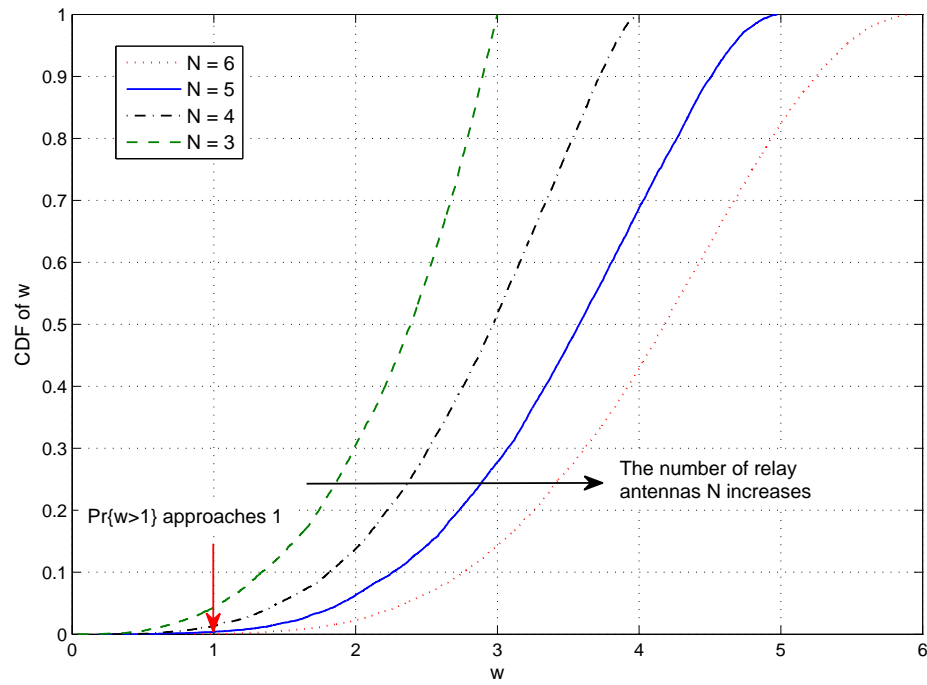


Fig. 1. System model.

Fig. 2. The CDF of w in Lemma 1.

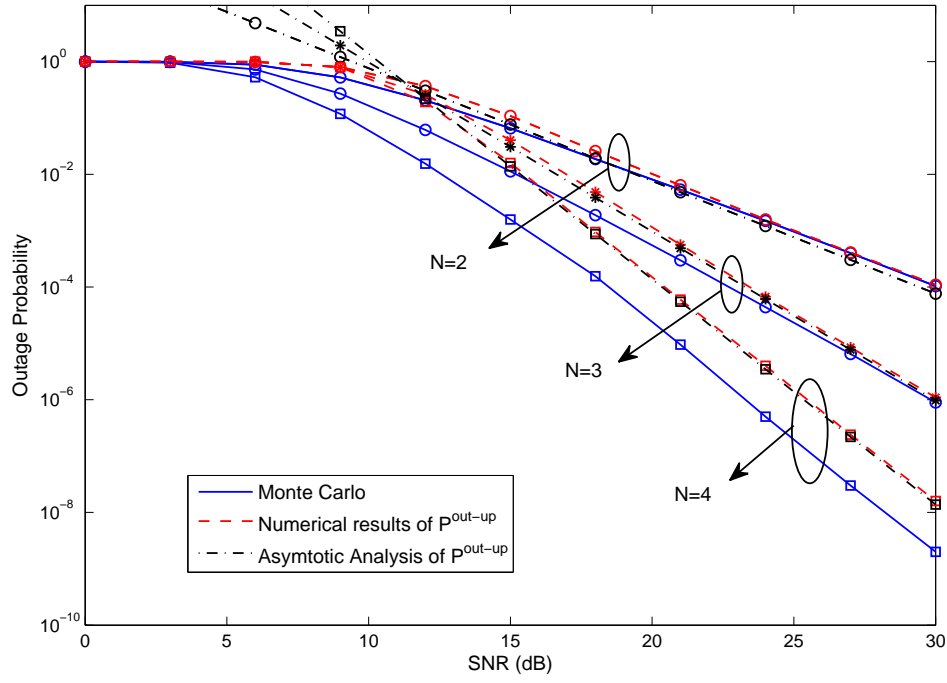


Fig. 3. The derived upper bound of outage probability, $N = 2, 3, 4$.

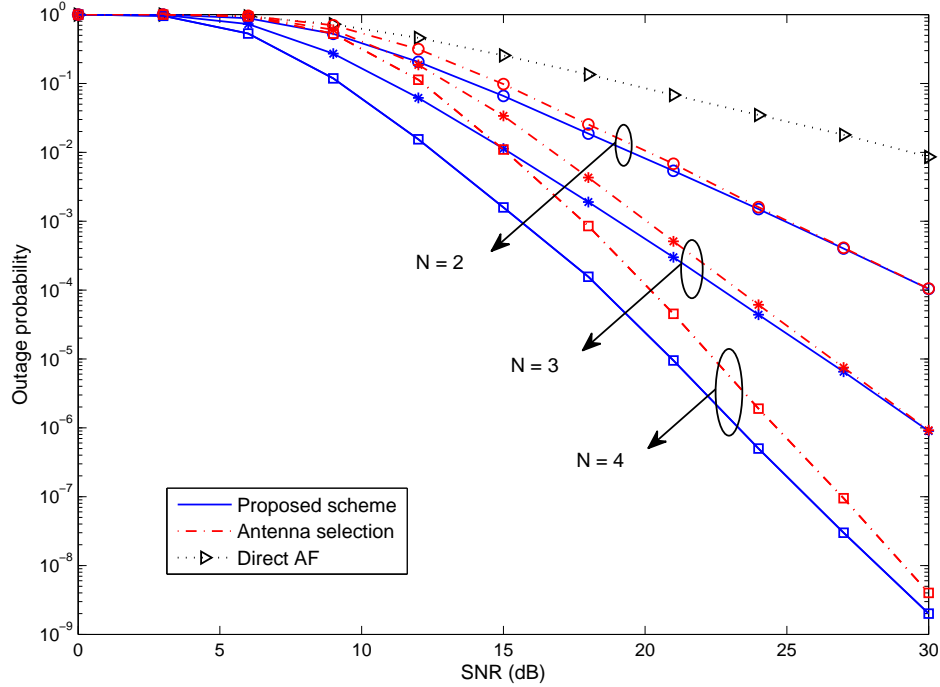


Fig. 4. The outage probability performance, $N = 2, 3, 4$.